1601/85

ACCOUNT OF AN OBSERVATION OF THE RIGHT ASCENSION AND DECLINATION OF MERCURY OUT OF THE MERIDIAN, NEAR HIS GREATEST ELONGATION, SEPT. 1786, MADE BY MR. JOHN SMEATON, F. R. S. WITH AN EQUATORIAL MICROMETER, OF HIS OWN INVENTION AND WORKMANSHIP; ACCOMPANIED WITH AN INVESTIGATION OF A METHOD OF ALLOWING FOR REFRACTION IN SUCH KIND OF OBSERVATIONS.

From the PHILOSOPHICAL TRANSACTIONS.

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A C C O U N T, &c.

Read at the ROYAL SOCIETY, June 7, 1787.

M. DE LA LANDE having announced to ſome of my aſtronomical friends the utility of accurate obſervations of Mercury, at his two elongations the laſt year, in Auguſt and September; I tried to get obſervations of that planet in croſſing the meridian, for ſome days before and after the greateſt elongation in Auguſt; and though the ſtate of the atmoſphere about that time was not very favourable to the purpoſe, yet there was one day that I thought unexceptionable, but could not perceive the leaſt appearance of Mercury; at which I was the rather ſurpriſed, as I had formerly ſeen that planet in the like ſituation, with the ſame inſtrument, with perfect perſpicuity*: and as I did not hear of any one elſe having ſucceeded in this obſervation, I thought it might be very poſſible for the ſame diſappointment again to happen, with reſpect to the approaching elongation in September. I judged, therefore, that it might be of ſome utility to aſtronomy, if, by *any means*, a good obſervation of Mercury could be got; and alſo, that it would be a proper ſubject whereon to make trial of an inſtrument for ſuch purpoſes, the idea of which I had conceived, and begun to conſtruct, above forty years before; but which, from various avocations, I did not perfect to my ſatisfaction till the year 1770*: ſince which time it has lain by, in hopes that ſomething might happen, by which a full and effectual trial might be made thereof.

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* The inſtrument mentioned is a tranſit, made by myſelf in the year 1768; at which time achromatic object-glaſſes not having been, ſo far as I knew, applied to aſtronomical inſtruments by others, but having found the good effects thereof for

This inſtrument was orginally intended as an improvement of the common wire micrometer, for the purpoſes of taking differences of right aſcenſion and declination, in a more commodious and effectual manner than could be done in the method then practiſed of uſing that inſtrument †; and at the ſame

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for other purpoſes, I reſolved to uſe a double achromatic object-glaſs, made by Mr. DOLLOND; which being of equal aperture with the ſimple object-glaſs, then in the tranſit of the Royal Obſervatory, this teleſcope I therefore eſteemed to be of nearly equal validity, as to quantity of light, with that at Greenwich, but reduced to the more commodious length of three feet ſix inches.

* Some obſervations made therewith, after it was completed, I tranſmitted to my friend Mr. AUBERT; the conſiſtency of which induced him to procure a ſimilar inſtrument to be made for *cometary* and ſuch kind of obſervations as cannot be commodiouſly made in the meridian. Of this inſtrument the Rev. Mr. WOLLASTON has made honourable mention in Phil. Tranſ. Vol. LXXV. for 1785, p. 348.

† The common wire micrometer, as uſed by Dr. BRADLEY, and deſcribed from a paper of the Doctor's hand-writing, is given by Dr. MASKELYNE in the Philoſophical Tranſactions, Vol. LXII. for the year 1772; and in addition to to which I muſt beg leave to obſerve, that the teleſcopes then in uſe for the micro-

ſame time more effectually to anſwer the purpoſe of GRAHAM's *aſtronomical ſector*, which was contrived by him (as Dr. SMITH informs us) to ſupply the deficiencies of the micrometer then in uſe *.

The moſt neceſſary and fundamental improvements ſeemed to be; firſt, that of rendering the micrometer teleſcope manageable upon an equatorial motion; and, ſecondly, the contrivance of a ſtand of ſuch ſolidity and ſtedfaſtneſs that the teleſcope might preſerve the poſition in which it was placed, for a length of time: for it occurred to me, that if the teleſcope could be maintained at reſt; or in a degree of ſtability ſuperior to that of the aſtronomical ſector; then the neceſſity of taking in a greater compaſs in declination than could be commodiouſly given to the field of a teleſcope would be the leſs neceſſary: for, inſtead of confining the object to a compariſon with a ſtar not differing more than a few minutes of time, or at moſt a quarter of an hour in right aſcenſion, thoſe compariſons could be extended to an hour or two, or even on occaſion to three or four hours; there being ſcarcely any part of the heavens ſo devoid of ſtars, of a ſuitable magnitude for theſe obſervations, but that a proper one may be found within that compaſs in right aſcenſion, provided there is allowed

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micrometer were from ten to fifteen feet long, made with wooden tubes, ſupported at each end upon two wooden ſupports, by which the teleſcope could be managed in altitude and azimuth; but not to follow a celeſtial object in its proper motion on one center: which apparatus, I believe, is ſtill remaining in the Royal Obſervatory.

[*] This inſtrument is deſcribed in SMITH's Optics, Vol. II. p. 350.; and the original one, made by Mr. GRAHAM, was, at his death, placed in the Royal Obſervatory, and is mounted upon a three-legged ſtand of *wood*.

the difference of a degree, either north or ſouth of the object, in declination [*].

Conſidering, however, that the approaching elongation would be in the morning; and that the beſt chance of ſeeing Mercury with this inſtrument would be ſome time in the twilight, between Mercury's riſing and the riſing of the Sun; yet, on ſuppoſition of catching the planet in his paſſage over the wires, there would be no chance of ſeeing any ſtar paſs over the field, wherewith to compare him, till the following evening, which being at leaſt fourteen hours, the certain poſition of the teleſcope for ſo great a length of time was *almoſt* more than I could reaſonably hope for. To judge how far I might form an expectation, by way of a previous trial, I compared Saturn with γ Capricorni [†], and found the return of the ſtar to the ſame place two evenings afterwards, both in right aſcenſion and declination, was ſo near, that I concluded I might very well expect a good obſervation of Mercury, in caſe I could get a ſight of him, though the ſtars wherewith he was to be compared lay at the diſtance of the following evening at the sooneſt.

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[*] The field of the teleſcope of Mr. AUBERT's inſtrument is two degrees; but that of the original, wherewith this obſervation was made, is 1° 17′: to gain which, the eye-glaſs being immoveable (and *achromatic* to prevent the indiſtinctneſs that would otherwiſe have taken place near the border) the magnifying power was obliged to be conſiderably reduced, in reſpect of what has been uſual for micrometers, that is, ſo as not to exceed 20 times: in conſequence, there is, therefore, no need for ſo long a teleſcope, this being but $34\frac{2}{3}$ inches focal length of object-glaſs; but being a double achromatic, made by the late Mr. JOHN DOLLOND, it is capable of as great an aperture as could be given to the ſimple object-glaſſes of twelve or fifteen feet teleſcopes, that were then generally given to micrometers; but the pencil of light being greateſt in this, is attended with this advantage, that the ſmall ſtars can be ſeen very diſtinct and in great abundance, like the modern night-glaſſes: and there is in reality no need of great magnifying powers for the preſent purpoſe; for the place of the wire being viewed by an eye-glaſs, of about $1\frac{3}{4}$ inches focus, its place may be diſtinguiſhed to leſs than a 2300th part of an inch, which, on the radius of $34\frac{2}{3}$ inches, is ſcarce $2''\frac{1}{2}$ of a degree; and which, as I apprehend, is nearer the truth than can reaſonably be expected from inſtruments out of the meridian.

[†] According to this obſervation ♄ preceded γ ♑ the 2d of Sept. at 9 h. 15′ P.M. mean time by 30′ 9″.7 MT, and with greater declination ſouth than γ by 41′ 23″.

The micrometer is furniſhed with five horary wires, denominated in their order *a*, A, B, C, D (B being the middle horary wire), and the two declination wires are denominated *A* and *B*, each moveable by a ſeparate and independent micrometer-ſcrew, from the outſide of the field to the center, and a little beyond it; ſo that each wire can be moved into the place of the other when at or near the center *.

The morning of the 23d of September, about a quarter paſt five o'clock, the air being clear and perfectly ſerene, it being then about an hour after Mercury's riſing, and near three-quarters of an hour before the riſing of the ſun, I very readily found Mercury with the teleſcope, and when found could eaſily ſee him with an opera glaſs; and Mercury being then in a ſtate of very little alteration of declination, I adjuſted one of the declination wires to his apparent run, by making him traverſe the *whole* field. The obſervations were then taken as in the firſt table; and in the evening I was lucky enough to get thoſe of λ Ceti and *o* Tauri, intending to repeat the whole the next morning and evening. The next morning proved cloudy, and ſo continued, that I ſaw the planet no more; but in the evening of the 26th, I found the ſtars come again ſo near the ſame declination, that I was encouraged to continue the obſervation to ſee what change would happen. It then came on bad rainy weather till the 30th, when I again repeated the obſervation, and found the ſtars to come ſo near in declination that I was fully ſatisfied of the ſtability of the inſtrument, ſo far at leaſt as could regard twenty-four hours: but as I was then appointed to go a journey, and could have no other uſe for the inſtrument, I locked the door of the Obſervatory, leaving the inſtrument in its poſition, that I might ſee what change would happen by the time of my return; and was quite aſtoniſhed to find, on the 13th of October, that it had remained in a manner unmoved; for it had ſuffered no more apparent alteration than what might occur by the errors of obſerving, and alterations of the clocks and tranſit.

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* In Dr. BRADLEY's Paper it is ſaid, that before the *late* alterations, both the declination wires were made moveable; and that it was an improvement to make one of them fixed, and one only moveable. But however they might be immediately preceding the Doctor's time, I believe, the original micrometers by Mr. TOWNLEY were with one fixed and one moveable declination wire, as I have ſeen one in this form among the remaining apparatus of Mr. ABRAHAM SHARPE. In an inſtrument, however, fitted up for the purpoſes of the *equatorial micrometer*, I believe, it will be found moſt convenient to have both thoſe wires moveable; as by this means they not only are enabled to ſlide into *each other's place*, but every part of the frame of the inſtrument remains fixed during the whole of the obſervation, the two ſlides carrying theſe two wires excepted.

It muſt, however, be remarked, that, beſides that in the conſtruction of the inſtrument, every thing was contrived that appeared likely to give it firmneſs; it was reſted upon the *fruſtum* of an hexagonal pyramid of ſtone, in the founding whereof, great care was taken as to its ſolidity; and was detached from the floor for ſupporting the obſerver.

This Obſervatory at *Auſthorpe* I eſteem in the latitude of 53° 47′ 54″ N. and 5′ 50″ of time W. from *Greenwich*.

Table I. Obſervations of Mercury at his elongation Sept. 1786, with an Equatorial Micrometer.

Day, object; and wires.	Hour.	Time as taken by the clock.	Time reduced to min. & ſec.	Reduced to the middle wire.	Mean of the wires.	Parts of the microm.	Micrometer reduc.
Sept. 23.	AM.	M. q. bea.	′ ″	′ ″	′ ″	Rev.Pts.	Rev.Pts.
Merc. to wire *a*	5	24 3 5	24 47.5	26 34.8			
A	—	25 3 14½	25 52.3	26 34.8			
Middle wire B	—	26 2 9½	26 34.7	26 34.7	26 34.7	*B* 28.85	S 0.74
C	—	27 1 6	27 18	26 34.7	N. B. The teleſcope's center was pointed to horary circle VI. 34½ Decl. N. 7° 48′.		
D	—	28 1 15	28 22.5	26 34.6			
	PM.						
λ Ceti to B	9	15 1 27	15 28.5	15 28.5	15 28.4		
C	—	16 0 23	16 11.5	15 28.3			
o Tauri to B	—	40 1 25	40 27.5	40 27.5	40 27.4	*B* 8.39	N 19.72
C	—	41 0 21	41 10.5	40 27.3			
Sept. 26.							
λ Ceti to *a*	9	2 2 13	2 36.5	4 23.5			
A	—	3 2 23	3 41.5	4 23.9	4 23.8	*B* 16.97	N 11.14
C	—	5 0 14½	5 7.3	4 24.1			
o Tauri to A	—	28 2 21	28 40.5	29 22.9			
B	—	29 1 16	29 23	29 23	29 23	*B* 8.47	N 19.64
Sept. 30.							
λ Ceti to *a*	8	47 0 3½	47 1.8	48 48.8			
A	—	48 0 13	48 6.5	48 48.9			
B	—	48 3 8	48 49	48 49	48 49	*B* 16.97	N 11.14
C	—	49 2 4½	49 32.3	48 49.1			
o Tauri to A	9	13 0 11	13 5.5	13 47.9			
B	—	13 3 6	13 48	13 48	13 48.1	*B* 8.48	N 19.63
C	—	14 2 3	14 31.5	13 48.3			
α Orionis to *a*	11	41 3 12	41 51	43 38			
A	—	42 3 20½	42 53.3	43 37.7			
B	—	43 2 17	43 38.5	43 38.5	43 37.9	*A* 15.07	S 15.77
C	—	44 1 12	44 21	43 37.8			
D	—	45 1 20½	45 23	43 37.7			
Oct. 13.							
λ Ceti to A	7	58 3 0	58 45	59 27.4			
B	—	59 1 25½	59 27.7	59 27.7	59 27.5	*B* 16.97	N 11.14
C	8	0 0 21½	59 10.7	59 27.5			
o Tauri to C	—	25 0 20	25 10	24 26.8	24 26.8	*B* 8.50	N 19.61
α Orionis to *a*	10	52 2 0½	52 30.2	54 17.2			
A	—	53 2 9	53 34.5	54 16.9			
B	—	54 1 4½	54 17.2	54 17.2	54 17.1	*A* 15.07	S 15.77
C	—	55 0 0½	55 0.2	54 17			
D	—	56 0 10	56 5	54 17.4			
1	2	3	4	5	6	7	8

Table II. For reducing the horary wires of the Equatorial Micrometer to that of the middle, when taken in mean ſolar time.

	Wires.	Equatorial object.		Declination 7°48′.	
		⊙'s run	✱'s run	⊙'s run	✱'s run
		′ ″	′ ″	′ ″	′ ″
The 1ſt wire precedes the midddle, add	*a*	1 46.2	1 46	1 47.3	1 47
2d ——— ———	A	1 42.1	1 42	1 42.5	1 42.4
3d, or middle wire – –	B	– –	– –	– –	– –
4th is ſubſequent to the middle, ſubtract	C	1 42.9	1 42.8	1 43.3	1 43.2
5th ——— ———	D	1 46.8	1 46.6	1 47.9	1 47.6
1	2	3	4	5	6

Table III. Containing the obſervations of Tab. I. reduced ſo as to ſhew the correct differences of right aſcenſion and declination between Mercury and the ſtars wherewith he was compared.

1786. Date and object.	Hour.	Paſſage over mid. hor. wire by journ. clock.	Correction to reduce the clock to mean time.	Correct mean time of the obſervation.	Intervals of mean time of different obſervations.	Parts of micrometer from the teleſcope's center.	Pts. of micr. reduced = declin. from the teleſcope's center.
Sept. 23.	AM	′ ″	′ ″	h. ′ ″		Rev.Pts.	′ ″
☿ to the mid. wire	5	26 34.7	−3 59.8	5 22 34.9		S 0.74	S 1 8
	PM				h. ′ ″		
λ Ceti to mid. wire	9	15 28.4	−4 0.1	9 11 28.3	15 48 53.4		
ο Tauri to the ſame	9	40 27.4	−4 0.1	9 36 27.3	0 24 59	N19.72	N 30 26
Sept. 26.							
λ Ceti to mid. wire	9	4 23.8	−4 43.2	8 59 40.6		N11.14	N 17 11
ο Tauri to the ſame	9	29 23	−4 43.2	9 24 39.8	0 24 59.2	N19.64	N 30 18
Sept. 30.							
λ Ceti to mid. wire	8	48 49	−4 50.9	8 43 58.1		N11.14	N 17 11
ο Tauri to the ſame	9	13 48.1	−4 50.9	9 8 57.2	0 24 59.1	N19.63	N 30 17
α Orion. to the ſame	11	43 37.9	−4 50.8	11 38 47.1	2 29 49.9	S 15.77	S 20 20
Oct. 13.							
λ Ceti to mid. wire	7	59 27.5	−6 36.4	7 52 51.3		N11.14	N 17 11
ο Tauri to the ſame	8	24 26.8	−6 36.4	8 17 50	0 24 58.7	N19.61	N 30 15
α Orion. to the ſame	10	54 17.1	−6 37	10 47 40.1	2 29 50.1	S 15.77	S 20 20
1	2	3	4	5	6	7	8

Table IV. Deviations in the direction of the axis of the teleſcope of the Equatorial Micrometer in right aſcenſion and declination in 20 days, from the 23d of September to the 13th of October, both incluſive.

Objects obſerved.		3 days from the 23d to 26th.	4 days from the 26th to 30th.	7 days from the 23d to 30th	13 days from the 30th to 13th.	17 days from the 26th to 13th	20 days from the 23d to 13th.
λ Ceti	R. Aſcenſion	0″	too late 1.1″	too late 1.1″	too ſoon 0.1″	too late 1.0″	too late 1.1
	Declination	not taken the 23d	exact 0.	not taken the 23d	exact 0.	exact 0.	not taken the 23d
ο Tauri	R. Aſcenſion	too late 0.2	too late 1.0	too late 1.2	too ſoon 0.5	too late 0.5	too late 0.7
	Declination	South 8.	South 1.	South 9.	North 2.	North 3.	South 11.
α Orionis	R. Aſcenſion	– – –	– – –	– – –	too ſoon 0.3	– – –	– – –
	Declination	– – –	– – –	– – –	South 5.	– – –	– – –

N. B. The right aſcenſion is expreſſed in the integer and decimal parts of a ſecond of time.
The declination is expreſſed in ſeconds of a degree.

Expla-

Explanation of the leſs obvious parts of the Tables of the Obſervation of Mercury *near his Elongation,* Sept. 1786.

The third column of Tab. I. contains the times of obſervation as they were taken down from the half-ſecond *journeyman* clock, in minutes, quarters, and *beats*, according to the following method; which was, by taking up the beat when the ſecond hand came to 15, 30, 45, or 60, and then counting 30 beats repeatedly till the arrival of the object at the middle of the wire it was approaching; after its arrival, the beats (or interval between two beats) being retained in memory, and the eye caſt upon the dial-plate, it was eaſily ſeen whether it was ſo many beats more than the quarter, the half, three-quarters, or the whole minute, and was ſet down accordingly. Thoſe reduced to minutes, ſeconds, and tenths of ſeconds, by allowing .3 for the quarter ſecond, .5 the half, and .7 for the three-quarters of a ſecond, are contained in the fourth column. The reduction of the fourth column to the fifth was by means of the auxiliary Tab. II.; and Mercury being then nearly ſtationary reſpecting the ſun, the ſun's run was uſed for the planet inſtead of that of a ſtar. The mean of each ſet of obſervations of the fifth column are carried into the ſixth.

The ſeventh column contains the parts of the micrometer as they were read off; to render which intelligible, it is to be noted, that the declination wire *A*, travels from the upper ſide of the field of view of the teleſcope towards the center, and ſomewhat beyond it; and upon it are taken all the objects that paſs the field of view on the upper ſide, anſwerable (by inverſion of the object) to the ſouthern half of the field: and in like manner thoſe that paſs the field of view on the lower half are taken upon the wire *B*, and for the ſame reaſon denote a declination north. The ſcale of the micrometers of each wire begins from a point aſſumed ſomewhat without the field, and the number increaſes from thence towards the center of the field, and continues beyond it; the integral parts are the turns of the ſcrew, and the centeſimal the diviſions of the index plate, being divided into 100 parts. The point of the ſcale, anſwerable to the center of the field of view, having been found by obſervations on each ſcale reſpectively; when the wire *A* (*Auſtralis*) ſtands at 30.84, it is in the center of the field; and when the wire *B* (*Borealis*) is at 28.11, it alſo cuts the ſame center. Hence the parts of the micrometer being reſpectively taken from thoſe two numbers (which may therefore be called *conſtant* numbers) the remainder will be the diſtance of each reſpective wire from the center in parts of the micrometer. Thus, in the obſervation of *o* Tauri upon the 23d, the parts are *B* 8.39; this taken from 28.11, leaves N 19.72, which are placed in col. 8. as the diſtance, in parts of the micrometer, that *o* Tauri paſſed north of the center of the field of view, or axis of the teleſcope.

In like manner, in the obſervation of Mercury on the 23d, the parts are *B* 28.85; but this being greater than the conſtant number 28.11, the exceſs will be .74 parts; which being the parts reaching beyond the center, they will be ſo much *ſouth* of it, and are ſet down therefore in col. 8. S 0.74: and in this manner the declinations of the reſt are made out, from their reſpective numbers of parts of the micrometer, and ſet down in col. 8.

The numbers of the ſixth column of Tab. I. are transferred to the third column of Tab. III.; and the declinations ſet down in parts of the micrometer, Tab. I. col. 8. are transferred to col. 7. of Tab. III.

Col. 4. of this table contains the corrections of the times deduced from the journeyman clock (as *per* col. 3.) to reduce it to mean time; which corrections are made out from the general account of the goings of the tranſit clock, corrected by tranſits of the ſun, taken the 22d, 23d, 27th, and 30th of September, and the 12th, 13th, and 14th of October *. The journeyman clock was regularly compared at nights and mornings with the tranſit clock; and generally immediately after the obſervation. The *meridian* and *rotative* obſervatories in which the clocks reſpectively were, are at the diſtance of 53 yards E. and W.; the compariſons were made by a ſeconds ſtop watch †.

The numbers of the fourth column being properly applied to thoſe of the third produce the fifth; and which, with the ſixth column, will be ſufficiently explained by their titles. The parts of the micrometer in the ſeventh column, being reduced into minutes and ſeconds, are contained in col. 8. and reſpectively ſhew the minutes and ſeconds at which each object paſſed to the north or ſouth of the center of the teleſcope. The value of the parts of the micrometer were obtained by previous obſervations, from whence the following rule was deduced: the numbers of turns and centeſimal parts being conſidered as integral, and divided by 1.08, the quotient will be the number of ſeconds. Thus, in the obſervation of *o* Tauri upon the 23d, the parts 1972, divided by 1.08, gives 1826″ = 30′ 26″; and the parts of mercury .74, divided by 1.08 = 68″ = 1′ 8″. Now the teleſcope being fixed to one point of the heavens during the whole period of theſe obſervations, without any motion of any of the parts, the ſcrews commanding the declination wires *A* and *B* excepted, we are enabled to judge of its ſteadineſs to this point by the following remarks. If it varied in declination, this would be ſhewn by the paſſage of the ſame ſtar at a different diſtance from the center of the teleſcope at different revolutions; and if it varied in right aſcenſion, it would be ſhewn by its not paſſing the horary wires at the due time, according to the acceleration of the ſtars upon the mean time of the ſun. Both the right aſcenſion and declination may be varied by differences of refraction of the air at the ſame altitude; and the right aſcenſion is further liable to be *apparently* varied, by the errors of the tranſit inſtrument, the tranſit clock, the transferring of its time to the journeyman clock, the intermediate errors of the ſame, and of the obſervation itſelf; and as there paſſed an interval of almoſt 16 hours betwixt the paſſage of Mercury over the field of view of the teleſcope and that of λ Ceti, which was the neareſt ſtar wherewith a compariſon could be made, it will be a ſatisfaction to ſee, as before intimated, what variations aroſe in ſtill greater intervals of time.

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* The tranſit clock was made by HINDLEY, and has a pendulum rod of *cedar* wood.

† The journeyman clock was generally ſet to the tranſit clock on Sunday mornings; and when from home the former was ſuffered to go down. The journeyman will *generally* agree with the tranſit clock to 2″ in 24 hours; but during the period of theſe obſervations, went remarkably well.

In right aſcenſion.

		h.	′	″
Thus λ Ceti upon Sept. 23. paſſed the horary wires at	–	9	11	28.3
and ———— 26. ————	–	8	59	40.6
λ Ceti therefore came ſooner in three days by	–		11	47.7
but ——— ought to accelerate on mean time –	–		11	47.7
——— therefore came after three days exactly to the time.				

Again,

	h.	′	″
Again, ο Tauri upon Sept. 23. paſſed the horary wires at -	9	36	27.5
and ——————— 26. ——————— -	9	24	39.8
ο Tauri therefore came ſooner after three days by - -		11	47.5
——— ought to accelerare on mean time - -		11	47.7
——— therefore came too late in three days by - -			.2

In declination.

	′	″
ο Tauri upon Sept. 23. paſſed north of teleſcope's center -	30	26
——————— 26. ——————— -	30	18
——— therefore paſſed leſs north, or more ſouth, than before by		8

In like manner every compariſon that Tab. III. affords is particularly ſet down in Tab. IV. which containing thirteen compariſons in right aſcenſion and ten in declination, the greateſt deviation in right aſcenſion is 1″.2, and 11″ of a degree in declination. This ſuppoſes every error before mentioned to reſide in the inſtrument, and every other inſtrument and obſervation, which were concerned in the reſult, to be perfect; which, from the ſmallneſs of the total errors, ſeems to indicate a degree of ſteadineſs in the inſtrument unexperienced or unnoticed before.

Deduction of the poſition of Mercury *from the preceding obſervations as ſet down in* Tab. III.

In right aſcenſion from col. 6.

	′	″
ο Tauri followed λ Ceti Sept. 23. - - -	24	59
——————— 26. - - -	24	59.2
——————— 30. - - -	24	59.1
——————— Oct. 13. - - -	24	58.7
——————— at a mean of the four - -	24	59

α Orionis

	h.	′	″
α Orionis followed *o* Tauri Sept. 30. - - -	2	29	49.9
——————— Oct. 13. - - -	2	29	50.1
——————— at a mean - - -	2	29	50
Now Mercury preceded λ Ceti Sept. 23. - -	15	48	53.4
λ Ceti preceded *o* Tauri by mean of four - -		24	59
o Tauri preceded α Orionis by mean of two -	2	29	50
Mercury therefore preceded α Orionis by - -	18	43	42.4

In declination from col. 8.

	′	″
Sept. 23. A.M. Mercury paſſed the middle horary wire, ſouth of its center	1	8
Same evening *o* Tauri paſſed the middle horary wire, north of it -	30	26
Therefore Mercury paſſed the middle horary wire more S. than *o* Tauri by	31	34

	′	″		′	″
But Sept. 26. λ Ceti paſſed N. of center	17	11	Diff.	13	7
——— *o* Tauri ———	30	18			
——— 30. λ Ceti ———	17	11	——	13	6
——— *o* Tauri ———	30	17			
—— Oct. 13. λ Ceti ———	17	11	——	13	4
——— *o* Tauri ———	30	15			

From the ſmallneſs of the above differences we may infer, that very little uncertainty in declination had attended the paſſage of *o* Tauri upon Sept. 23.

	′	″		′	″
Upon Sept. 30. *o* Tauri paſſed N	30	17	Sum	54	37
——— α Orionis —— S	24	20			
Upon Oct. 13. *o* Tauri paſſed N	30	15	——	54	35
——— α Orionis —— S	24	20			

	′	″
α Orionis then at a mean paſſed more ſouth than *o* Tauri -	54	36
Merc. therefore on the 23d paſſed with more N. declination than α Orionis	23	2

Inveſtigation

Inveſtigation of the effects of refraction.

The preceding deductions and remarks ſhew the conſiſtency of the obſervations with themſelves; yet, from the poſition of the teleſcope, it being only elevated 11°½ above the horizon *, it is neceſſary to examine how far the deductions above ſpecified were capable of being affected by refraction. And in this reſpect it will appear, that if it be ſuppoſed, there is no difference in the quantity of refraction of ſuch objects as appear within the limits of the field of view of this inſtrument (which is 1° 17′), then their relative poſitions to each other will not be affected thereby: for if in fig. 1. (Tab. XIII.) we ſuppoſe the circle VHRO to repreſent the boundary of the field of view, HO being an horizontal and VR a vertical line, each paſſing through the center of the field at L; and if PLP denotes a part of a parallel of declination, then BLX perpendicular thereto, will be a part of an horary circle, both paſſing through the ſame center. Now let $d*$ be the apparent path of a ſtar, ſuppoſing it unaffected by refraction till it comes to the vertical line at $*$, and there to be lifted up by refraction in the ſaid vertical to L. Let $e+$ denote another ſtar, alſo unaffected by refraction, to paſs along the different parallel of declination $e+$ till it comes to +; then, if it be ſuppoſed that the two ſtars are both ſituated in the ſame horary circle, if at the point + refraction takes place, and by hypotheſis this is lifted up equally with the other, in the perpendicular $+l$, then the line $+*$ being drawn through the places of the two ſtars, will be cotemporary and parallel to LX; and the figure $l+*L$ being evidently a rhomboides, the two ſtars, ſo altered by refraction, will arrive together at the horary circle LX at the ſame time, and with the ſame difference of declination, as if no refraction had taken place. It is therefore only the *difference* of refraction which takes place in objects at different heights in the *ſame field*, that can alter their relative ſituations: however, it appears neceſſary to examine what this may amount to.

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* This will readily be deduced by inſpection of the celeſtial globe.

Let the letters in fig. 2. denote the ſame things as before; to which we will add, that *a*, A, B, C, D, denote the parallel horary wires of the micrometer, and *AA*, *BB*, the declination wires, denoted *A* and *B* in the tables: now from the celeſtial globe we ſhall alſo readily obtain the horary angle VLP = 54°½ = L*bc*. Let now an object paſs along the wire *AA* from the horizontal line at *d* to the vertical line at *b*; in this it will paſs through a difference of refraction, according as it gets more and more elevated above the horizontal line HO; and let the elevation L*b* be half a degree or 30 minutes: then, according to Dr. BRADLEY's Table of Refraction*, the difference of refraction betwixt the 78th and 79th degrees of zenith diſtance is 23″.6, half of which 11″.8, may be eſteemed the difference of refraction for a difference of half a degree of altitude at 78°½ zenith diſtance, or of 11°½ altitude: the object, therefore, in paſſing from the horizontal line at *d* to the vertical line at *b* paſſes through every difference of refraction from 0″ to 11″.8; and the queſtion is, how much it is at a *medium*, that is, when it arrives at the middle wire at the point *c*? From this point let fall the perpendicular *ce*. Now, the proportion of the ſides of the triangle *db*L being given from conſtruction, they may be taken off by a ſcale, *viz.*

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* Inſerted in Dr. MASKELYNE's Obſervations, Vol. I. p. 15.

Suppoſe L*b* = 174
db = 299
*d*L = 242
and aſſuming the ſide L*b* = 30
the other ſides by proportion as above will be { *db* = 51.6
*d*L = 41.7

The triangles L*bc* and *dce* are ſimilar to *db*L; therefore ſay, as *db* = 51.6 : *d*L = 41.7 :: L*b* = 30 : L*c* = 24; and as L*b* = 30 : L*c* = 24 :: *d*L = 41.7 : *dc* = 33.5; and again, as *db* = 51.6 : *dc* = 33.5 :: L*b* = 30 : *ce* = 19.5: but this will affect the declination, only in proportion of the line *ef* drawn parallel to LX; and it will affect the right aſcenſion according to the line *fc*: but the triangle *ecf* being ſimilar to the original one *db*L, we ſhall have *db* = 51.6 : L*b* = 30 :: *ce* = 19.5 : *fc* = 11.3 for the line affecting the right aſcenſion; and alſo, as *db* = 51.6 : *d*L = 41.7 :: *ce* = 19.5 : *ef* = 15.8 for the line affecting the declination. But the effect of difference of refraction upon the line L*b* = 30′ being only 11″.8, the reſpective effects of the lines *fc* and *ef* will be in proportion; that is,

as 30′ : 11′.3 :: 11″.8 : 4″.4 for the effect in right aſcenſion,
and as 30 : 15.8 :: 11.8 : 6.2 ———————— declination;

but as it has been determined, that when the line L*b* is 30 minutes, the line LC, or the correſponding declination, will be only 24 minutes; the effects of refraction above ſtated will be therefore due to 24′.

Correction for the poſition of the wires.

The above corrections take place on ſuppoſition that the ſeveral wires of the micrometer were ſtrictly parallel to the reſpective parts of the circles of declination, and horary circles in the heavens; but in the practical uſe of this inſtrument it is found more convenient, on account of a ready and certain adjuſtment, to place one of the wires AA or BB parallel to the apparent track of the ſtar wherewith the *planetary* body is to be compared: in conſequence, when the ſtar ∗, fig. 1. is lifted up to L, it will not ſtrictly purſue the line LP; but being leſs and leſs lifted up as it mounts higher, it will apparently fall more and more below the line LP as it aſcends above the line HO, and will therefore take a courſe, ſuppoſe Lp. The wire PLP being therefore adjuſted to agree with pLp; by conſtruction of the inſtrument, the wire BLX will aſſume the poſition qLx perpendicular to pLp. The ſtar, therefore, that ran along the parallel $e+$ before it ſuffered refraction, and at + was ſuppoſed to be lifted up to l, there not meeting LX will take the courſe ly, nearly parallel to Lp, and have ſome diſtance, as lz, to travel before it arrives at the new-placed wire Lx; and it is now proper to examine what this quantity may be.

Through the point z draw the line $rzos$ parallel to HO, and cutting the vertical RLV in o, and let Lo be aſſumed = 30′; then, ſince the angle XLx is ſuppoſed to be *minute*, the *groſs* proportions of the ſides of the triangles Lyz and Lyl may be, for this purpoſe, ſuppoſed the ſame[*], and the ſame as Lbc, dbL, fig. 2. to which the triangle Lzo, fig. 1. will alſo be ſimilar; as likewiſe the triangle yzo, and alſo the little triangle zlv: but making the ſide lv of the triangle zlv equal to the effect of refraction in perpendicular = 11″.8; then, to find the ſide lz, the diſtance run from the firſt to the laſt ſuppoſed place of the wire, we need only ſay, as $Lb = 30 : db = 51.6 :: lv = 11''.8 : lz = 20''.3$; and this will be its value when the declination Lc, fig. 2. is $24'$; but then the declination Ll or Lz, fig. 1. being greater than the perpendicular ſide Lo (aſſumed $30'$) in the proportion of $Lz : Lo$, ſay, by ſimilarity of triangles converſely, as $dL = 41.7 : db = 51.6 :: Lo = 30' : Lz = 38'.2$; but as the correction before ſtated of $20''.3$ is an angular error, taking place in proportion to the diſtance from the center, or the declination; for the declination given of $24'$ ſay, as $38.2 : 24' :: 20''.3 : 13$; to which adding $4''.4$, we ſhall have $17''.4$ for the whole error in right aſcenſion, ſuppoſing it in the equator, but muſt be again increaſed in the proportion in which a ſtar having declination is ſlower than a ſtar in the equator; that is, it muſt be increaſed in the proportion of any of the numbers in the fourth column of Tab. II. to the ſimilar ones in col. 6. of the ſame table; that is, as $1'\,46'' : 1'\,47''$ or as $106 : 107, :: 17''.4 : 17''6$ *.

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[*] I am aware, that the ſuppoſition of the ſides of the triangles Lyz and Lyl being the ſame cannot be ſtrictly ſo; nor can they have the ſame proportions; nor are any of the lines concerned right lines, that are ſuppoſed ſuch; but aſſumptions *near* the truth are allowable for the correction of an error in the *greateſt part*, that if uncorrected would ſcarcely amount to a *groſs error*.

As all theſe errors, ariſing from difference of refraction, are in proportion of the diſtance of the object from the center of the teleſcope, they will take place in proportion to the difference of declination of the two objects to be compared, whether they have paſſed the field on the ſame, or on different ſides of the center. Now the difference of declination of Mercury and α Orionis being only $23'\,2''$, and the quantities being made out for $24'$ ſay (rejecting the 2 ſeconds), as $24' : 23' :: 17''.4 : 16''.7$, which turned into time in the run of the ſtar will be $1''.1$ in right aſcenſion.

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* My friend Dr. MASKELYNE obſerves, that in *ſtrictneſs* each ſtar ought to have its own proper reduction, on account of *difference* of declination, which in *extreme caſes* will amount to a ſenſible quantity.

Say again, as 24′ : 23′ :: 6″.2 : 6″, the correction in declination. From the near equality of the lines L*l* and L*z*, it is evident, that no correction of *declination* is neceſſary on account of the inclination of the wires, the whole difference falling in right aſcenſion. As therefore Mercury paſſed with 23′ 2″ more north declination than *α* Orionis, and paſſed through a part of the *medium* that lifted him up leſs; it therefore gave him leſs north declination than it did to *α*, and therefore apparently diminiſhed the real difference; hence 6″ muſt be added to the apparent difference 23′ 2″, making it 23′ 8″ difference of declination: and as Mercury was lifted up leſs than *α*, he would not ſo ſoon come to the middle wire by 1″.1 as he ſhould have done, he therefore came too late by 1″.1, which muſt be ſubtracted from the time of Mercury's paſſage the 2d of Sept. which will increaſe the time in which he preceded *α* Orionis; that is, 18 h. 43′ 42″.4 increaſed by 1.1 will become 18 h. 43′ 43″.5 difference of right aſcenſion.

I have been the more particular in the inveſtigation of this obſervation, firſt of all to aſcertain the degree of dependance that may be formed on an inſtrument of the kind; and, ſecondly, to infer ſuch eaſy and ſimple rules, that other ſimilar obſervations may be the more eaſily reduced. Being therefore ſatisfied of the ſtability of the inſtrument; if we had concluded the obſervation with that of Mercury in the morning, and of *o* Tauri in the evening of the 23d, then the reſult from Tab. III. ſhould have been

	h.	′	″
Mercury paſſed the wires at – – –	17	22	34.9
And *o* Tauri paſſed at – – –	9	36	27.3
Difference of right aſcenſion – – –	15	13	52.4

which is the very ſame as was before deduced from the mean of the whole:

		′	″
And if to Mercury's declination ſouth of teleſcope's center	–	1	8
We add *o* Tauri's ——— north ———	–	30	26
We ſhall have for the difference of declination	– –	31	34

the ſame as before determined. Our obſervation would, therefore, in this caſe ſimply have been, that Mercury preceded *o* Tauri in right aſcenſion 15 h. 13′ 52″.4 mean time, and paſſed the wire with more ſouth declination than *o* Tauri by 31′ 34″.

After this, *o* Tauri would have required to be compared with ſome well rectified ſtar by meridian inſtruments; but in the preſent caſe α Orionis, one of Dr. MASKELYNE's Catalogue of 34 principal ſtars happened to lie ſufficiently near the ſame parallel of declination, to admit of *o* Tauri to be compared therewith by the ſame inſtrument, while pointed to the ſame place of the heavens. The operations which were ſubſequent, therefore, muſt be conſidered as intended to ſave thoſe of a meridian inſtrument.

Now had our obſervation concluded with the above, then the correction would have taken place upon the difference of declination of *o* Tauri with Mercury, inſtead of the ultimate one with α Orionis; but it muſt be obſerved, that whatever quantity of correction the difference of declination would occaſion, it would be compenſated in the difference of refraction of of *o* Tauri and α Orionis, when they came to be obſerved on the meridian; however, in the preſent caſe it happens to be more commodious, as both can be done under one.

Preparatory then to the laying down the ſimple rule for the correction of refraction, it is proper to premiſe, that it is evident,

dent, the lines, fig. 2. Lb, Lc, ce, ef, being in continued proportion Lb will be to ef in triplicate proportion of Lb to Lc; and that Lc will be to ef in duplicate proportion of $Lb : Lc$. The difference of declination, therefore, due to 30′ difference of elevation will be as Lb to Lc ſimply; but the effect of difference of refraction in declination will be leſs than the difference of declination in the proportion of $Lb^2 : Lc^2$; and that the effect of difference of refraction in right aſcenſion will be leſs than the difference of refraction in declination in the proportion of $Lb : cb$ ſimply.

Now it has been remarked, that the elevation of the teleſcope's center above the horizon, and the horary angle VLP, will always be readily given near enough for the purpoſe by the globe. A triangle given Lbd can therefore be conſtructed, and the ſide Lb being made 30′ (or any convenient aliquot part of a degree) the other ſides will be found by proportion: ſay then, as in the preſent caſe, $db = 51.6 : dL = 41.7 :: Lb = 30 : Lc = 24$, for the difference of declination correſponding to half a degree of altitude: ſay then, as $51.6^2 : 41.7^2$, that is, as $2663 : 1739 :: 24 : 15.7 = ef$. But without troubling ourſelves with high numbers, if we take the proportion 51.6 to 41.7 by the ſlide-rule *twice*, we ſhall arrive at 15.7, near enough for the value of the line ef: ſay then, as $Lb = 30 : ef = 15.7 :: 11''.8 : 6''.2$ for the refraction in declination: and as $dL = 41.7 : Lb = 30 :: 6''.2 : 4''.4$ for the refraction in right aſcenſion, according to the *true* poſition of the wires: and, for the correction of right aſcenſion in the poſition of the wires, ſay,

$$\text{As } \overbrace{Lb = 30 \quad : db = 51.6}^{\text{Fig. 2.}} :: \overbrace{lv = 11''.8 : lz = 20''.3}^{\text{Fig. 1.}};$$

$$\text{and again, } dL = 41.7 : db = 51.6 :: Lo = 30' \quad : Lz = 38'.2.$$

Take now L*b*, fig. 1. = L*c*, fig. 2. = 24′, and draw the line *hik*, fig. 1. parallel to the line *lzy*, and then ſay, as Lz = 38′.2 : L*h* (or L*i*) = 24′ :: *lz* = 20″.3 : *hi* = 13″, which + 4″.4 = 17″.4 for the whole error in right aſcenſion, with a declination or diſtance from the center of 24′; but as the errors both of right aſcenſion and declination are in proportion to diſtance from the center, as the difference of the planet and ſtar is only 23′, ſay, as 24′ : 23′ :: 17″.4 : 16″.7 = 1″.1 time; and for the declination, ſay again, as 24′ : 23′ :: 6″.2 : 6″ declination *.

Reduction of Mercury's *compariſon with* α Orionis, *to right aſcenſion and declination.*

We have laid it down, that the 23d Sept. 1786, A.M. at 5 h. 22′ 34″.9 mean time, Mercury preceded α Orionis 18 h. 43′ 43″.5, and had then a more northern declination by 23′ 8″.

According to Dr. Maskelyne's Catalogue of 34 ſtars, the right aſcenſion of α Orionis reduced to the time when he was obſerved is 85° 54′ 12″.

Now as the whole circle of the ſphere makes a revolution in the time that α Orionis makes one turn, which is

h. ′ ″
23 56 4.1 then from this deduct
18 43 43.5
5 12 20.6 remains for the time that α Orionis preceded

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* If the compariſon had been with *o* Tauri, then we muſt have ſaid,

As 24′ : 31½′ :: 17″.5 : 23″ = 1″.5 of time,
and 24 : 31½ :: 6.2 : 8.1 correction for declination.

N. B. All theſe and the above proportions will be commodiouſly wrought with the ſlide-rule.

Mercury in right aſcenſion; but if α ran the whole rotation = 360° in 23 h. 56′ 4″.1, what portion of it will be run in 5 h. 12′ 20″.6 = 18740″.6?

But 24 h. = 86,400 ſeconds, and 360 = 1,296,000 ſeconds:

		°	′	″
Time. Time. : Of degrees. Of degrees Say then, as 86400″ : 18740″.6 :: 1296000″ : 281109″ =	–	78	5	9
But, according to Dr. MASKELYNE's ſelect Catalogue, the right aſcenſion of α Orionis for Sept. 30, 17[illegible] was, (which add)	–	85	54	12
The right aſcenſion of Mercury at the time of obſervation was therefore		163	59	21

			°	′	″
According to Dr. MASKELYNE's ſelect Catalogue α Orionis had declination north, corrected for preceſſion	–	–	7	21	8.8
The ſum of aberration and nutation from *Connoiſſance des Temps*		–			+ 8.4
The correct declination north of α Orionis	–	–	7	21	17.2
To which add that Mercury paſſed more north	–	–		23	8
Mercury's declination therefore was	–	–	7	44	25.2

The reſult.

1786, Sept 23. A.M. at 5 h. 22′ 35″ M.T.	Mercury's	right aſcenſion	–	163	59	21
		declination north	–	7	44	25

BRITISH MUSEUM

Philos. Trans. Vol. LXXVII. Tab. XIII. p. 342.

Fig. 2.

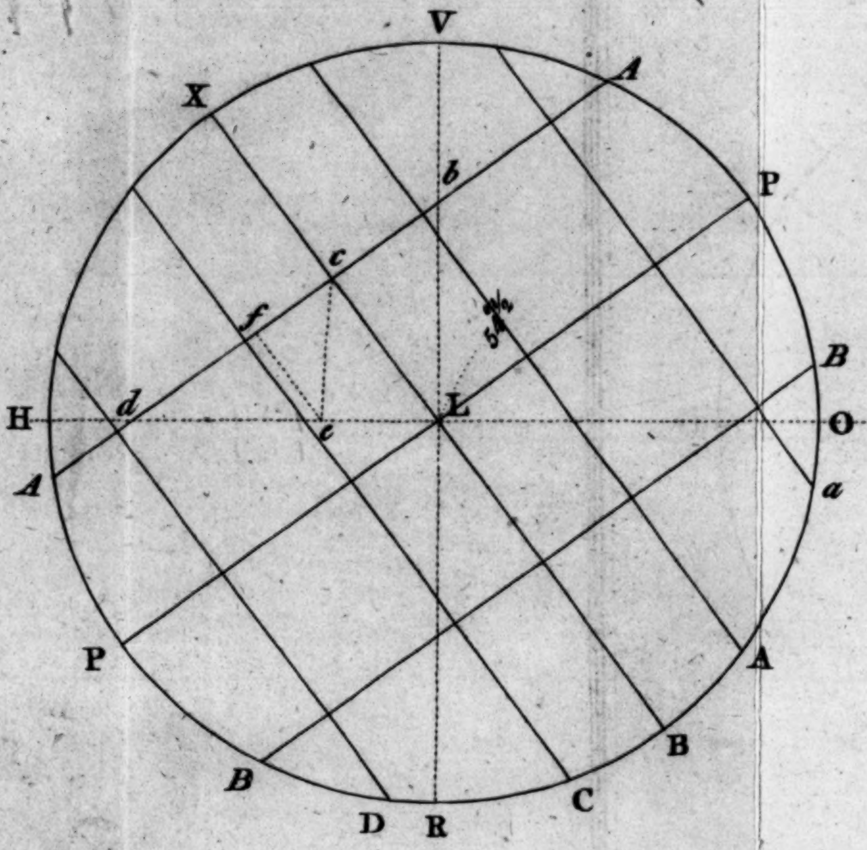
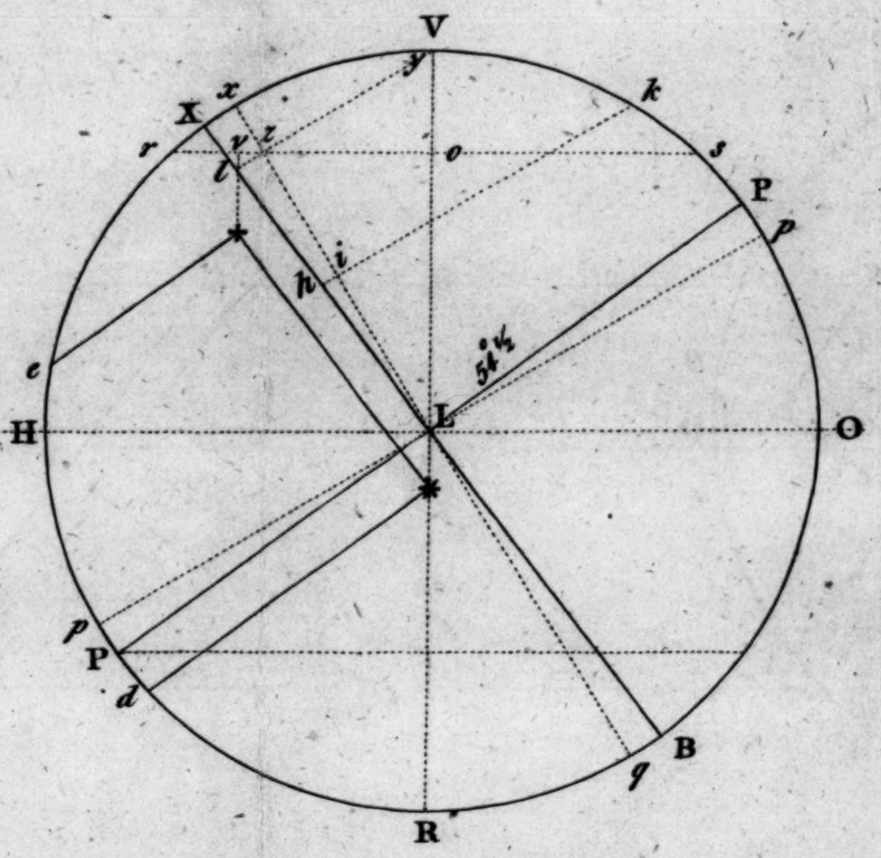


Fig. 1.



Basire Sc.